Geostatistical Simulation of Permeability Tensors

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The geological heterogeneity at all scales within a reservoir presents a challenge for reservoir characterization. More specifically, flow simulation is infeasible for models built with a high resolution. There is a need to assign effective parameters at a relatively coarse scale that represent features at smaller scales. Unstructured grids are being increasingly used to discretize the reservoir into unequal grid blocks that maintain higher resolution in some regions of interest. Reservoir properties must be upscaled to the unstructured grid cells. Permeability is the most difficult. This paper gives a workflow for upscaling permeability; the end result is a tensor property. Since flow simulation is used to acquire the tensors, processing time may still be impractical for large grids. Some alternative methods are discussed that may prove more efficient in upscaling permeability.

Introduction

Flow simulation on large grids may require a significant computer and professional resources. A method to overcome this issue is representing the impractically large flow system with another that is less computationally demanding. Unstructured grids are being targeted in this paper as the coarse scale system used to summarize high resolution information including facies, porosity and permeability. Scalar permeability must be upscaled to a set of permeability tensors, which can be used in flow simulation algorithms with the unstructured grid.

One possible workflow for upscaling high resolution data to an unstructured grid for further processing is discussed in this paper. In general, this involves generating fine scale models of facies, porosity and permeability with cosimulation, then upscaling those properties to the unstructured cells in an appropriate manner. Facies may be represented as proportion curves and porosity as an arithmetic average. However, permeability requires flow simulation to recover tensors. Permeability tensors are needed to capture the effects heterogeneity throughout a coarse cell has on fluid flow. Only single phase flow is considered in this paper, although multiphase flow will be mentioned.

Computational processing demands for upscaling permeability to coarse cells may be impractical. Because permeability is treated as a random function, there may be a large set of realizations that require upscaling. This, combined with a grid comprising many cells, could result in impractical computational expense. The use of alternative methods to flow simulation for characterising an unstructured grid with permeability tensors is motivated.

Background

Unstructured grids for solving reservoir flow simulation studies are being targeted because of their flexibility in representing geological features of the reservoir and in creating high resolution near production and injection wells. Flow simulation on high resolution grids, which are initially used in an attempt to capture all heterogeneities and detail in a reservoir, is an intractable problem. The flow system is much too large to solve with conventional computing power. As a result, the high resolution model must be upscaled to a coarser one, which ideally will lead to the same flow response. Upscaling permeability still requires flow simulation at the fine scale, but with subsets of the full high resolution model. This form of upscaling is a *local* method (Farmer, 2002). Subsets may cover an individual unstructured cell or a set of cells. Local upscaling can still be a time consuming process especially when dealing with large unstructured grids. Some subsets may contain tens of thousands of high resolution observations for one unstructured cell.

Permeability tensors are a component of several of the formulae used to approximate flow through porous media. For example, Darcy's law (Equation 1) in three dimensions involves a permeability tensor **K**, the pressure gradient, ∇p (∇ is the 3D gradient operator), the gravitational constant, g, a unit vector pointing down, **h**, and the fluids density and viscosity, ρ and μ. Equation 1 defines a volumetric flux vector **q**. Permeability tensors consisting of 9 elements (Equation 2); however, they are typically assumed diagonal in most reservoir flow simulator software. They can also be symmetric to account for any anisotropy in permeability. The case of non-symmetric tensors, referred to as full tensors here, occurs only when dealing with non-linear boundary conditions.

$$\mathbf{q} = \frac{-\mathbf{K}}{\mu}\left(\nabla p - \rho g \mathbf{h}\right) \tag{1}$$

$$\mathbf{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \tag{2}$$

For high resolution models of permeability, the assumption that permeability tensors are diagonal and even isotropic is acceptable: any anisotropic behaviour of the medium is accounted for by the model as a whole, or by subsets of it. Tensors for unstructured grid cells will have to capture any anisotropic behaviour of the subset they encompass. Consequently, coarse-scale permeability tensors may be symmetric or full.

Upscaling Workflow

A process for characterising an unstructured grid with permeability tensors is developed. Each cell contains information about a reservoir including facies, porosity and permeability. It is assumed that this information is at a higher resolution than the unstructured cells. Different upscaling methods are used for each data type: proportion curves for facies, arithmetic mean for porosity and flow simulation for permeability. The upscaling workflow is broken down into two parts: (1) high resolution property modeling and (2) upscaling. The first is discussed as a geostatistical reservoir modeling process while the second will cover upscaling steps for the three data types: facies, porosity and permeability.

High resolution modeling

As the focus of this paper is on upscaling permeability tensors, this section will not impart a high level of detail. References including Deutsch (2002), Chiles and Delfiner (1999) and Isaaks and Srivastava (1989) can provide more detail of the components discussed here. Initial components to any geostatistical modeling application are data acquisition, analysis and cleaning, declustering, variography including indicator variograms for facies and normal score transformation. Once this pre-processing is completed, estimation and simulation of target variables is carried out. This section is concerned with the simulation of facies and cosimulation of porosity and permeability. The domain of interest (D) is one that encompasses an unstructured grid that is to represent a reservoir, but partitioned at a much higher resolution.

Variables such as porosity and permeability are modeled dependent on the type of facies they were sampled from. Statistical characteristics of these continuous variables, such as their distribution and variography, are often different within different facies types. Facies are thus modeled first in the workflow. There are several geostatistical algorithms available to do this including indicator simulation and truncated Gaussian simulation. These will generate a set of equally probable realizations of facies within D.

Simulation of porosity and permeability is done by facies and for each facies realization. Typically, these two variables show a high correlation and realizations are generated with Gaussian cosimulation, the underlying estimator being collocated cokriging. Resulting models should be validated. There are several tools available for this including histogram and variogram reproduction, cross validation, cross plots for multiple variables, and accuracy plots.

Upscaling

Upscaling of facies, porosity and permeability to unstructured grid cells is done to provide a final model that is amenable to large scale reservoir flow simulation analysis. Facies were already used to condition porosity and permeability when generating high resolution models. They are also required for relative permeability calculations in multiphase flow regimes. For single phase flow, facies are required only to generate the permeability field and are not needed for the unstructured grid.

Upscaling of facies is an odd concept. Unlike straight forward averaging calculations for continuous variables, facies cannot be summarized in this way (as an average category). The *most-of* operation is one possible technique where an unstructured cell would be represented by a single facies category. For a particular facies realization, all high resolution values within an unstructured cell would be involved in the calculation. The most abundant category would be assigned to the cell. This method destroys much of the information that was obtained with high resolution modeling. More information can be captured by saving facies proportions within each unstructured cell in the form of a density function or cumulative density function. This would be used to average relative permeability curves.

Porosity is upscaled to unstructured cells with an arithmetic average (Bear, 1972). By generating porosity models on a regular grid and at a sufficiently fine scale, it can be assumed that each one represents an equal volume. Thus, arithmetic averaging can be done in an equal-weighted fashion. Other forms of upscaling such as summation (pore volume) would be valid as well. Porosity is useful for conditioning permeability values during the modeling process, calculating saturations for phases involved in flow, and in subsequent volumetric calculations with the unstructured grid such as oil-in-place.

Permeability cannot be upscaled with simple operators like facies and porosity if one is to represent the effects it has on flow through unstructured grid cells. An individual high resolution permeability value can be assumed isotropic, as can its effect on flow. A set of permeability values, i.e. those within an unstructured cell, will undoubtedly contain heterogeneities and flow will be anisotropic. Combinations of high resolution permeability will cause flow to divert in various directions and this type of behaviour is represented with a tensor (Equation 2).

Permeability Tensor Calculation

There are two primary steps in calculating permeability tensors for unstructured grid cells: (1) choose a subset of high resolution permeability and (2) flow simulate the subset. The subset may be chosen to either encompass an individual cell, a cell with an additional buffer region, or a set of unstructured cells (Figure 1). The use of a buffer region has been advocated to produce more accurate results (Wen, Durlofsky, and Edwards, 2003) than one fit tightly around an unstructured cell.

In this paper, calculating permeability tensors for unstructured cells was accomplished using a finite-difference flow solver. It is also possible to use different flow systems such as finite elements and finite volumes. Regular gridded high resolution diagonal permeability that encompasses an individual unstructured cell is used to setup the finite difference equations. Boundary conditions are selected to impose pressure gradients throughout the subset and a solution is found for the volumetric flux. By evaluating a series of different boundary conditions, an over-determined system can be developed from which a diagonal, symmetric or full permeability tensor can be calculated (Holden and Lia, 1992; Wen, Durlofsky and Edwards, 2003). Ignoring the gravitational term in Equation 1 and assuming unit viscosity, minimization of Equation 3 gives the effective permeability tensor, $\tilde{\mathbf{K}}$, where $\mathbf{q}$ is the volumetric flux using flow simulation and N is the number of boundary conditions evaluated.

$$E = \sum_{k=1}^{N} \left(\mathbf{q}_k - \tilde{\mathbf{K}} \nabla p_k \right)^2 \tag{3}$$

Constraints can be imposed on Equation 3 to give the desired form of tensor. Unconstrained, a full permeability tensor will result. Non-symmetric tensors have been proven to occur by upscaling (Wouter, 1996). To the knowledge of the author, they are not used in reservoir simulation software. Flow simulation results on coarse unstructured grids with full permeability tensors should be more accurate relative to the results obtained were the high resolution permeability model used instead. This is intuitive as full tensors will better capture the heterogeneity of the high resolution permeability within a coarser cell.

This same procedure could be applied over a subset of unstructured cells. Pressure differences and volumetric fluxes are known at all high resolution points from flow simulation. Calculating ∇p and $\mathbf{q}$ using only those high resolution points within a specific unstructured cell are used in Equation 3 to solve for that cell's tensor.

Example

A small unstructured grid was constructed and used in the above workflow to populate the cells with permeability tensors. Execution time was recorded to set a benchmark for this workflow and motivate other alternatives (Table 1). The unstructured grid comprised 99 3D voronoi cells spaced over three layers with 33 cells per layer (Figure 2). Porosity and permeability were cosimulated on a high resolution regular grid consisting of 2 million blocks (Figure 3). Block counts in the x, y and z direction were 200, 200 and 50 respectively. Fifty realizations were created.

Table 1: Execution times

Operation	Characteristics	Execution Time
High resolution cosimulation of porosity and permeability	2 million blocks, 50 realizations	1h 34m 10s
Permeability upscaling (flow simulation) and tensor calculation	99 cells, 50 realizations → 4,950 tensors	13h 53m 51s

Calculating permeability tensors was done on a cell by cell basis with no buffer region around the cells. They were calculated for all cells and all realizations for a total of 4,950 tensors. Execution time of this effort is recorded in Table 1. The distribution of sub-model sizes used in flow simulation indicates an average flow system size of 12,700 blocks (Figure 4). Sub-models ranged from 1,089 to 26,240 blocks.

Flow simulation of the sub-models was done with a steady-state, single phase, finite difference solver. 20 boundary conditions were evaluated for each unstructured cell to create the over-determined system for Equation 3. Three types of tensors were calculated from Equation 3: diagonal, symmetric and full. Diagonal tensors were solved as a linear regression problem. Symmetric and full tensors were solved using Newton method optimization which converged in the majority of cases in one step. The actual percentage of time to calculate tensors was insignificant with the time to solve flow equations over the sub-models.

Alternatives

Considering the execution time to determine permeability tensors for unstructured grid cells in the above example, which was a substantially small problem, other methods are motivated. Real reservoir modeling project would involve much larger unstructured grids. Larger high resolution models would be generated for upscaling and flow would likely be multiphase. In that case, using the above workflow to acquire tensors would only be feasible on small subsets of the full problem. Characterizing the full reservoir with tensors would not be possible. Other methods must be identified to achieve this goal.

Several possibilities exist for solving this problem. One is to infer spatial tensor distributions under a statistical framework. This involves incorporating statistical knowledge of a reservoir into the above workflow so that permeability tensors may be inferred for a full unstructured grid from a sub-sample of tensors determined by flow simulation. Another method involves enhancing aspects of the workflow to simplify computation. For example, other upscaling techniques and flow simulation techniques could be implemented to reduce processing time.

There are many reservoir parameters and variables that influence a permeability tensor. It may be possible to construct distributions and relationships between variables such that tensors could be inferred statistically for a cell rather than needing flow simulation to do so. Information that will affect the outcome of flow simulation for a particular cell includes the following:

- High resolution porosity distribution (statistical and spatial)
- High resolution permeability distribution (statistical and spatial)
- Facies proportions

- Cell geometry
- Flow conditions
- Information about neighbouring cells

Understanding how these relate to permeability tensors requires an initial sample of tensors, which may be calculated for a subset of unstructured cells using the above workflow. Remaining cells would be characterized by multivariate geostatistical techniques.

There are also several areas where the above workflow could be simplified to expedite calculations. Current methodology involves flow simulation of a high resolution regular grid of permeability. This is not a restriction. Other techniques such as finite element and finite volume flow simulation could be used on irregularly distributed permeability fields. This added flexibility would allow a more adaptive approach to the high resolution modeling phase. For example, unstructured cells could be subdivided by space partitioning algorithms to a set of points to characterize with scalar permeability. The number of points per cell could be controlled by a posteriori error bounds of the flow simulation technique being implemented. In the example, some cells were characterized with more than 20,000 points. Decreasing this number would greatly reduce processing time.

Conclusions

A workflow to calculate permeability tensors for an unstructured grid has been discussed. It is one possible solution to the problem of representing an intractable flow problem, that defined by a high resolution model, with a coarser system. The coarse scale system makes global reservoir flow achievable and ideally will give the same or similar results as the high resolution model. However, upscaling to the coarse model remains complex and the given workflow may become impractical for large models. Two possible directions to improve on the workflow have been identified, one delving into statistics and approximation, the other into more technical enhancements of the existing methodology. Determining the suitability of alternative methods to characterize an unstructured grid with permeability tensors should be done with global reservoir flow simulation studies. Ideal solutions may be obtained from flow simulation of a high resolution model.

References

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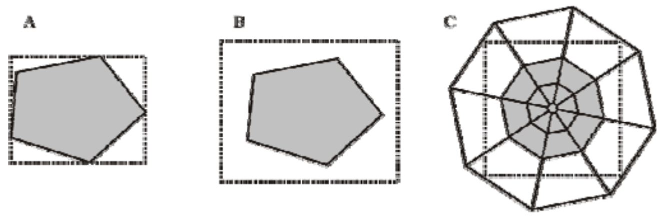

Figure 1: High resolution regions (dashed boxes) surrounding a cell exactly, A, with a buffer, B, and surrounding multiple cells, C. Highlighted cells are assigned a tensor after flow simulation.

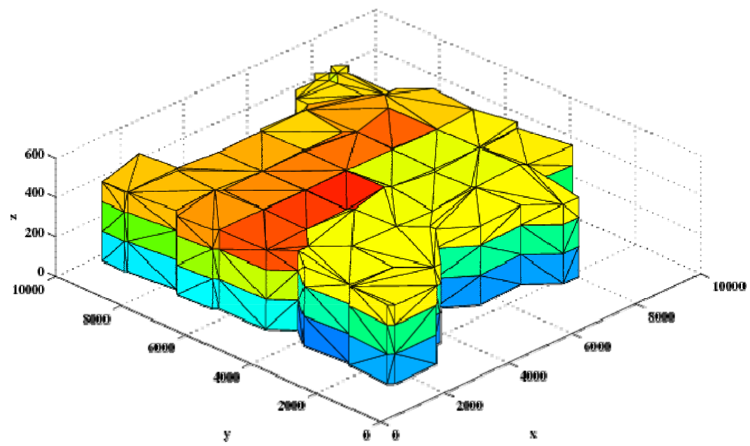

Figure 2: Unstructured (voronoi) grid shaded by cell index.

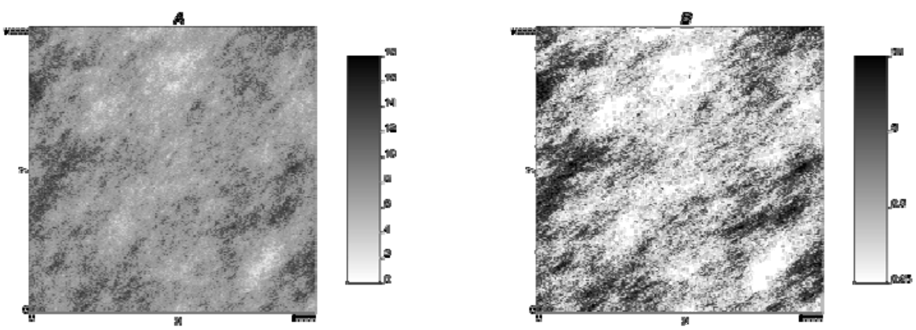

Figure 3: High resolution porosity, A, and permeability, B, for realization 5, layer 20.

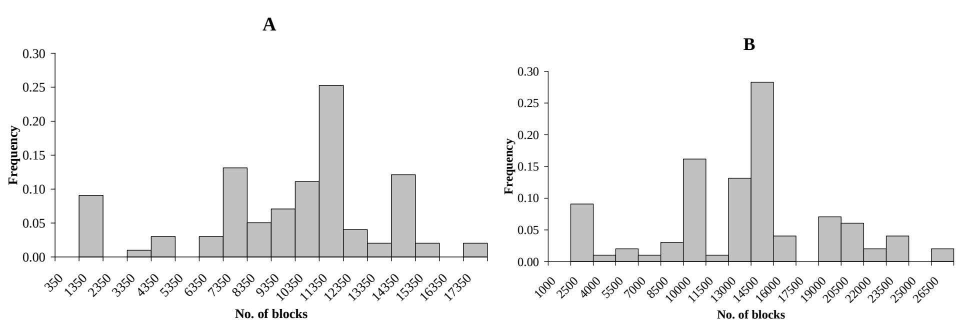

Figure 4: Distribution of high resolution blocks in unstructured cells, A, and flow model sizes, B.